

# Establishing Wear Particle Limits Using the Theory of Dynamic Equilibrium Condition (DEC) to Predict when Abnormal Wear Modes are Taking Place in Machinery

Thomas G. Barraclough and Daniel P. Walsh | AMETEK Spectro Scientific

## Synopsis

Extensive work on dynamic equilibrium models of wear particles was undertaken during the early 1980's to better help oil analysis professionals identify abnormal wear modes when performing analytical ferrography [1]. It was demonstrated that particles of different size ranges reach their own dynamic equilibrium condition based upon a balance between production and loss rates of wear particles in any given machine. Dynamic equilibrium condition (DEC) of particles, and the use of this concept is very helpful in setting wear particle limits for all types of machinery. With no pre-defined limits set by Original Equipment Manufacturers (OEMs), it is often difficult for the equipment owner or oil analyst to establish condemnation levels for machines generating large quantities of normal (benign) wear particles as a consequence of their normal operation. Understanding and using dynamic equilibrium models can help end users establish alarm limits for assets with unique operating profiles.

In this paper, two well-established mathematical models describing this condition are applied to real-world examples that show the time to equilibrium is not intuitively obvious. Understanding at what level dynamic equilibrium occurs is critical to understanding what is considered a normal or an abnormal wear rate for any given machine.

## Introduction

Machine condition has long been monitored by microscope examination of wear particles using analytical ferrography [2] and more recently by LaserNet Fines [3]. These invaluable techniques not only provide the oil analyst with the root cause of the abnormal wear mode but also provide detailed predictive information on any abnormal wear rate occurring in a machine. It has been demonstrated that when a machine enters an abnormal wear mode, the size and population of wear particles increases. In the case of a diesel engine, large severe sliding wear particles are produced when wear surface stresses become excessive due to load and/or speed [4]. Other critical rotating equipment such as gear drives, transmissions and wind turbines all produce abnormal wear that is distinctive relative to the mode of failure such as overload, sand/dirt abrasion, and lubrication starvation. Dynamic Equilibrium Condition (DEC) is defined as a steady state condition where the normal wear rate in a machine results in no net gain or loss of particles. Knowing what that level is in any lubrication system is necessary in order to detect departures from this condition as a result of an abnormality. In order to understand dynamic equilibrium, there are two models commonly referenced: Anderson-Driver and Kjer models. They are perhaps the best known and most relevant models for understanding particle generation behavior in lubrication systems.

### Anderson-Driver Model for Particle Equilibrium

Anderson and Driver [5] demonstrated how particles reach an equilibrium concentration by summing a geometric series based upon particle production and decay for a constant wear rate and given size. During passage of oil through the system (Figure 1), it is assumed that there are various mechanisms for the removal of particles:

- 1) Filtration
- 2) Settling
- 3) Impaction and adhesion
- 4) Comminution (grinding up of particles)
- 5) Dissolution (oxidation or other chemical attack)
- 6) Magnetic separation

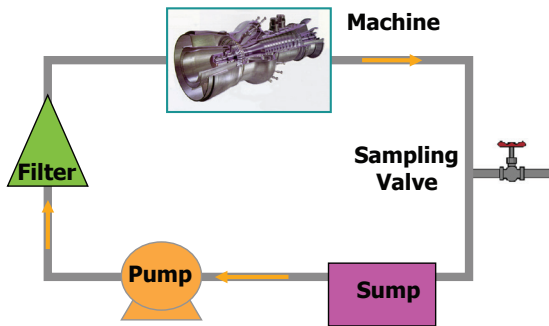


Figure 1: Simplified Oil Path.

The model shows how to sum the geometric series based on the net particle concentration in each cycle and arrive at a simple formula for calculating the time to come to equilibrium under various conditions. At equilibrium, the concentration can be calculated directly as in equation [1].

$$\text{Equilibrium Concentration (size)} = \frac{\text{Production Rate}_i}{\text{Removal Efficiency}_i} \quad [1]$$

This methodology is used to predict how many oil cycles occur before an equilibrium level is reached for a given particle size range.

This is of critical importance and a frequently asked question because it allows the equipment maintainer to know when is the correct time to begin taking representative samples after an oil change or engine overhaul.

In equation [2] the Anderson-Driver Model yields the number of cycles (R) to reach an equilibrium concentration for given particle loss coefficient ( $\beta$ ) and particle size ( $a_i$ ). If one assumes that if the ratio of a given particle concentration (after R cycles) to the final equilibrium concentration is  $> 1-\beta$ , where  $\beta \ll 1$  then the number of cycles will be close enough to the final equilibrium concentration.

$$R > \frac{\ln \beta}{\ln (1-a_i)} \quad [2]$$

For example, if  $\beta$  is chosen to equal 0.01 then R cycles will be calculated when the final equilibrium concentration is  $(1 - 0.01) = 99\%$  of the true equilibrium concentration.

Table 1 shows the number of cycles required to reach 99% of the equilibrium concentration for values of  $a_i$ .

NUMBER OF CYCLES	$A_i$ (PARTICLE LOSS COEFFICIENT FOR A GIVEN PARTICLE SIZE i)				
	0.8	0.1	0.05	0.01	0.001
$\infty$	1.25	10.0	20	100	1000
3	1.24				
44		9.903			
90			19.802		
459				99.007	
4603					990

Table 1: The number of cycles required to reach 99% of equilibrium concentration for given loss coefficients  $a_i$ .

### 1.1. Kjer Model for Particle Equilibrium

The Danish researcher Kjer [6] developed a model of particle equilibrium which demonstrates how the concentration of wear particles varies with time under different conditions and simplified assumptions. An exponential relationship between the concentration of particles for a given size versus time is derived by solving a linear first order differential equation based on particle balance.

The rate of increase in concentration of large particles is:

$$\frac{dC_L}{dt} = \frac{P_L - kC_L}{V}$$

Where:

- CL = Concentration of large particles
- PL = Production rate of large particles
- K = Removal Rate Constant
- V = Oil Volume
- t = time

Integrating from  $t=0$  and  $CL = 0$ , gives the concentration of large particles as a function of time.

$$C_t = P_L/k(1 - e^{-k(V)t})$$

### Using the models to determine limits

The best way to examine these dynamic equilibrium models is to consider an example such as a diesel engine: The predicted time it takes for an engine to reach a dynamic equilibrium level for a given particle size is calculated using both approaches. The relationship between particle concentration and time for a given particle size result are exponential for both models.

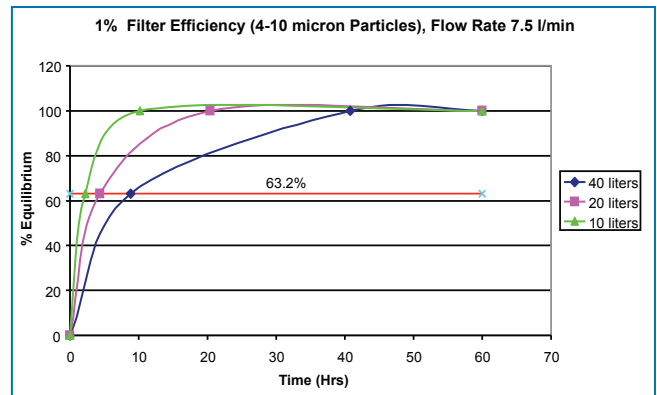
For this example, take a typical 40 micron in line engine oil filter that is common on diesel engines for highway applications. A multi-layer pleated filter will have a typical filter efficiency of 90 % or better at 40 microns, and we will expect a much lower capture efficiency of 1% for particles in the 4 to 10 micron range. The lube oil pump will deliver a constant flow rate of 7.5 liters/min through the filter. From this, we derive the following time to equilibrium relationships using the two models for different engine oil volume capacities, as illustrated in Table 2 and Figure 2.

Oil Volume (liters)	1% Capture Efficiency (4-10 micron particles)			Time to Equilibrium	
	FlowRate (L/hr)	# Cyc	Total vol through the filter	Anderson Driver Time (hrs)	Kjer Time (hrs) 63.2%
40	450	459	18360	41	9
20	450	459	9180	20	4.5
10	450	459	4590	10	2
5	450	459	2295	5	1

**Table 2:** Time to equilibrium for different oil volumes assuming a 1% capture efficiency and 7.5 liter / min filter flow rate.

This simple diesel engine example shows that oil volume capacity, oil flow rate, filtration efficiency and particle size *will all have an effect on the time to equilibrium*.

This example demonstrates that the time to equilibrium (no matter which model is applied) *is relatively fast even for the largest capacity engine*. However, this may take much longer in other applications where the filtration, oil volume and flow rates causes the time to equilibrium concentration to increase- as in slow speed bath lubricated gearboxes. The equipment maintainer will always have these multiple variables in the system, and will most often not have the time to estimate them and employ a model. So how do you use this knowledge?



**Figure 2:** Combined time to equilibrium models for different oil volumes derived from Table 2.

## 2. Establishing limits using DEC

The fundamental premise of machine condition monitoring by wear particle analysis is that an abnormal wear mode causes an increase in the size and concentration of wear particles above a previously defined baseline. Such limits can be set for the type of wear according to particle size ranges. The practical consequence of DEC is that if samples are taken periodically from a *normal running machine*, the concentration and size distribution of the wear particles should be about the same.

In order to determine alarm limits for a given machine, a dynamic equilibrium state needs to be established based on a minimum of five samples. The effect of an oil change must be considered because of the time it takes to regain the equilibrium particle concentration. The characteristic operating time it takes to return to dynamic equilibrium varies with the machine from which samples are taken, along with the other factors previously described.

### 2.1. Case Study

The following example shows this is a two step process: First a dynamic equilibrium baseline based on five or more samples is established. Second, alarm limits are calculated and established. Consider Figure 3 where a series of samples are taken on a machine. The first sample is high, indicating larger numbers of particles due to break-in wear. After the second sample, the machine appears to have reached a constant state of normal benign wear so a limit was set based on the equilibrium level as shown in Figure 3.

Alarm limits are established by first calculating the standard deviation (sigma [s]) of the sample range in the selected trend, and recording

it as a limit above the mean. The production of wear particles as measured is a normal distribution (i.e., the concentration is directly proportional to wear rate). Multiples of sigma can thus be used to establish the caution and warning limits.

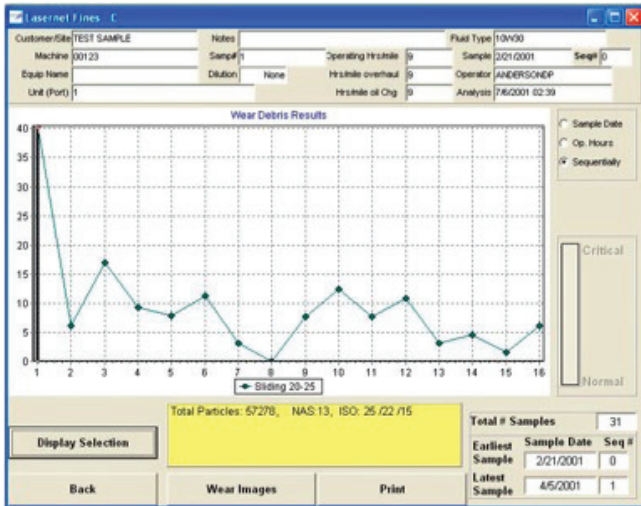


Figure 3: Normal benign wear sliding 20-25µm.

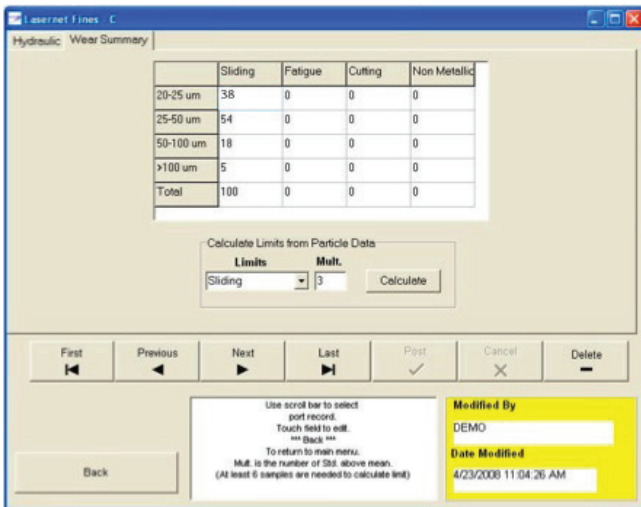


Figure 4: Calculation of Limit for given wear particle type.

The multiple of sigma determines the confidence factor on the number of particles that fall within the mean and the upper alarm limit. A one sigma value provides a 68.7% confidence factor that all the particles above this limit are due to an imminent problem and not due to normal wear. A two sigma value provides a 95.4% confidence factor and a three sigma value a 99.7% confidence factor. The standard deviation

multiple used depends on the machine being monitored and the desired confidence factor.

In this case study, a value of three sigma was used to calculate the limits for sliding wear particles in four size ranges. The three sigma value gives us 99.7% confidence that any particle concentration recorded above this limit sets off an alarm and any particles that are counted below this limit during sampling are treated as normal wear. The limit for the 20 - 25 µm range of sliding wear particles calculated by the software in this example was 38 particles (Figure 4).

The limit is depicted on a trend plot and the sample status bar is green indicating the sample is normal (Figure 5).

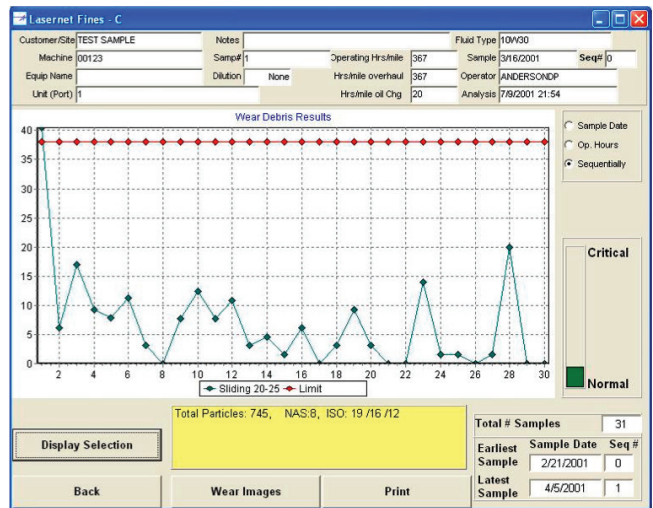


Figure 5: Normal sample in trend.

An increase in sliding wear particles exceeded the alarm limit (Figure 6) indicating that the wear rate in the machine had increased. The software warns the user that a critical level of wear has been reached.

## Conclusion

Particle behavior in a machine and dynamic equilibrium, although not intuitively obvious at first, is fundamental for the equipment owner to be able to predict when an abnormal wear mode is occurring. It also lets the oil analyst calculate when to take a sample between an oil change or engine overhaul based on certain operating parameters. Large particles need to be tracked closely because they are predominantly the first signs of an increasing wear rate and abnormal wear mode in a machine. Applications such as large diesel engines, wind turbine gearboxes, and complex lubrication systems can greatly

benefit from these concepts, which can not only measure the debris of interest, but set up abnormal alarms based on well-established scientific models for the dynamic equilibrium of wear particles.

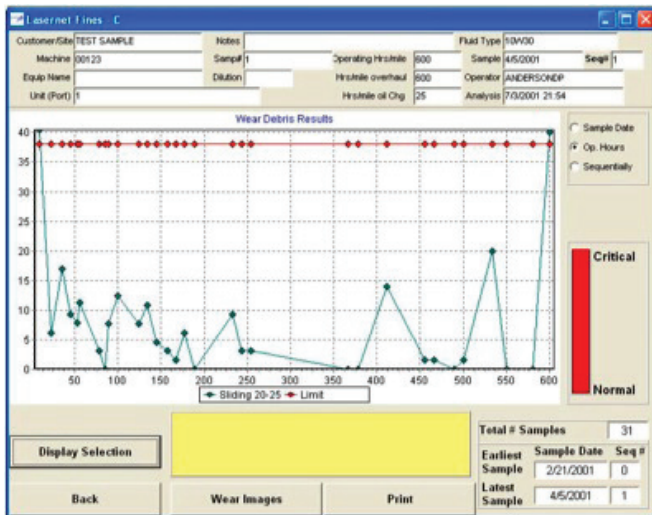


Figure 6: Abnormal machine state, Limit exceeded.

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